1 NUMBER SYSTEMS

Introduction

- **Number:** It means that something you would count.
- **Natural numbers:** They are the counting numbers, denoted by N.
  $\therefore$ N = {1, 2, 3, 4, 5, ...}
- **Whole numbers:** The natural numbers together with zero are called whole numbers and denoted by W.
  $\therefore$ W = {0, 1, 2, 3, 4, 5, ...}
- **Negative numbers:** The numbers that are opposite to the positive numbers are called negative numbers.
- **Integers:** It is a whole number (not a fractional number) that can be positive, negative or zero and denoted by Z.
  $\therefore$ Z = {..... –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5 .....}
  Note: Numbers like $\frac{3}{4}, \frac{1}{2}$, 1.15, 6.7 etc. are not integers.
  (*i*) Negative integers = {..... –6, –5, –4, –3, –2, –1}
  (*ii*) Positive integers = {1, 2, 3, 4, 5, 6 .....}
  (*iii*) Non-negative integers = {0, 1, 2, 3, 4 .....}
- **Rational numbers:** A number that can be written in the form $\frac{p}{q}$ is called a rational number, where p and q are integers and $q \neq 0$. It is denoted by Q.
  For example: If $p = 4$ and $q = 3$, then $r = \frac{p}{q} = \frac{4}{3}$ is a rational number.

Irrational Numbers

- **Irrational numbers:** A number that cannot be written in the form $\frac{p}{q}$ is called an irrational number, where p and q are integers and $q \neq 0$.
  For example: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$, 0.161016100161000161.....
  Note: (*i*) Zero is not an irrational number. It is a rational number.
  (*ii*) Surds are the irrational numbers.
- **Real numbers:** The collection of all set of rational and irrational numbers together are known as real numbers, denoted by R.
  There is a unique real number corresponding to every point on the number line. Conversely, corresponding to each real number, there is a unique point on the number line. Hence, number line is called real number line.

Real Numbers and their Decimal Expansions

- **Decimal expansion of rational number:** The decimal expansion of a rational number is either terminating or non-terminating recurring.
  For example: (*i*) 0.44444 .... = $0.\overline{4}$ (*ii*) 1.323232 ..... = $1.\overline{32}$
  (*iii*) 0.3525252 ...... = $0.3\overline{52}$ etc.

- **Decimal expansion of irrational number:** The decimal expansion of an irrational number is non-terminating non-recurring.

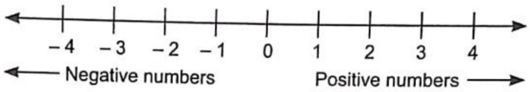
  For example: (*i*) $\sqrt{2} = 1.4142135623$ .... (*ii*) $\pi = 3.1415926535$ ....

  (*iii*) 1.202002000200002 .... (*iv*) 2.16016001600016000016 ....

Representing Real Numbers on the Number Line

Number line/Real line/Real Number line

- It is a horizontal line such that corresponding to every real number, there is a point on the real number line, and corresponding to every point on the number line, there exists a unique real number.
- Since zero is a real number, so corresponding to zero, there is a unique point on the number line called origin, and to the right of origin, all points are positive numbers, while left of this point, all points represent negative numbers.



- The point corresponds to real number with a terminating decimal expansion on the number line can be visualised by the process of sufficient successive magnification.

Operations on Real Numbers

Properties of irrational numbers:

- Like rational numbers, irrational numbers also satisfies the commutative, associative and distributive laws for addition and multiplication.
- The sum, difference, quotients and products of two irrational numbers are not always irrational.

  (*i*) $\sqrt{13} + (-\sqrt{13}) = 0$ (Rational) (*ii*) $(3+\sqrt{2}) + (3-\sqrt{2}) = 6$ (Rational)

  (*iii*) $\sqrt{5} - \sqrt{5} = 0$ (Rational) (*iv*) $\frac{\sqrt{12}}{\sqrt{12}} = \frac{2\sqrt{3}}{2\sqrt{3}} = 1$ (Rational)

  (*v*) $\sqrt{5} \times \sqrt{5} = 5$ (Rational)
- The sum or difference of a rational number and an irrational number is always an irrational number. For example:

  (*i*) $a + \sqrt{b}$ is an irrational number. (*ii*) $a - \sqrt{b}$ is an irrational number.
- The multiplication and division of a non-zero rational number with an irrational number is always irrational. For example:

  (*i*) $a\sqrt{b}$ is an irrational number. (*ii*) $a \div \sqrt{b}$ is an irrational number.

  (*iii*) $\sqrt{a} \div b$ is an irrational number.
- The multiplication and division of an irrational number by another irrational number results to a rational number. For example:

  (*i*) $5(\sqrt{3})^2 \div 4 = 5 \times 3 \div 4 = \frac{15}{4}$ (*ii*) $(\sqrt{3} - 2)(\sqrt{3} + 2) = (\sqrt{3})^2 - 2^2 = -1$

Operation of taking square roots of real numbers:

- **Surd:** Let $a > 0$ be a real number and n be a positive integer. Then $\sqrt[n]{a} = b$, if $b^n = a$ and $b > 0$. So, any number in the form $\sqrt[n]{a}$ and cannot be written as a rational number is called surd.
- The symbol $\sqrt{}$ is called radical sign.

- 'n' is known as order of surd and 'a' is known as radicand.
- Every surd is an irrational number, but every irrational number is not a surd.

Identities related to surds:

Let a and b be positive real numbers. Then the following identities holds:

(i) $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$ (iv) $(a+\sqrt{b})(a-\sqrt{b}) = a^2-b$

(v) $(\sqrt{a}+\sqrt{b})^2 = a + 2\sqrt{ab} + b$ (vi) $(\sqrt{a}-\sqrt{b})^2 = a-2\sqrt{ab}+b$

(vii) $(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d}) = \sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$

Rationalisation: The process of reducing a given surd to a rational form after multiplying it by a suitable surd is known as rationalisation.

For example: To rationalise the denominator of $\frac{1}{\sqrt{a}+b}$, we **multiply** this by $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$, where a and b are integers.

To rationalise the denominator of $\frac{1}{\sqrt{a}-\sqrt{b}}$, we multiply this by $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$, where a and b are integers.

The rationalising factor of $\frac{1}{a \pm \sqrt{b}}$ is $a \mp \sqrt{b}$.

Laws of Exponents for Real Numbers

- **Laws of Exponents:** Let m and n be exponents (powers) of base 'a' and $a > 0$. Then

  (i) $a^m . a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $\frac{a^m}{a^n} = a^{m-n}, m > n \left[\because \frac{1}{a^n} = a^{-n}\right]$

  (iv) $a^{-m} = \frac{1}{a^m}$ (v) $a^0 = 1$ (vi) $a^m b^m = (ab)^m$

  (vii) $(a^m)^{-n} = a^{-mn}$ (viii) $\frac{a^{-m}}{a^n} = a^{-m-n} = a^{-(m+n)}$ (ix) $a^{-m} \times a^{-n} = a^{m+n}$

  Here, the base is a positive real number and the exponents are rational numbers.
- Let $a > 0$ be a real number. Let p and q be integers such that p and q have no common factor other than 1 and $q > 0$.

  Then, $(a)^{\frac{p}{q}} = (a^p)^{\frac{1}{q}} = \sqrt[q]{a^p}$ or $a^{\frac{p}{q}} = \left(a^{\frac{1}{q}}\right)^p = (\sqrt[q]{a})^p$

  So, both operations are possible.
- **Extended laws of exponents:** Let $a > 0$ be a real number and m and n be rational numbers. Then, we have

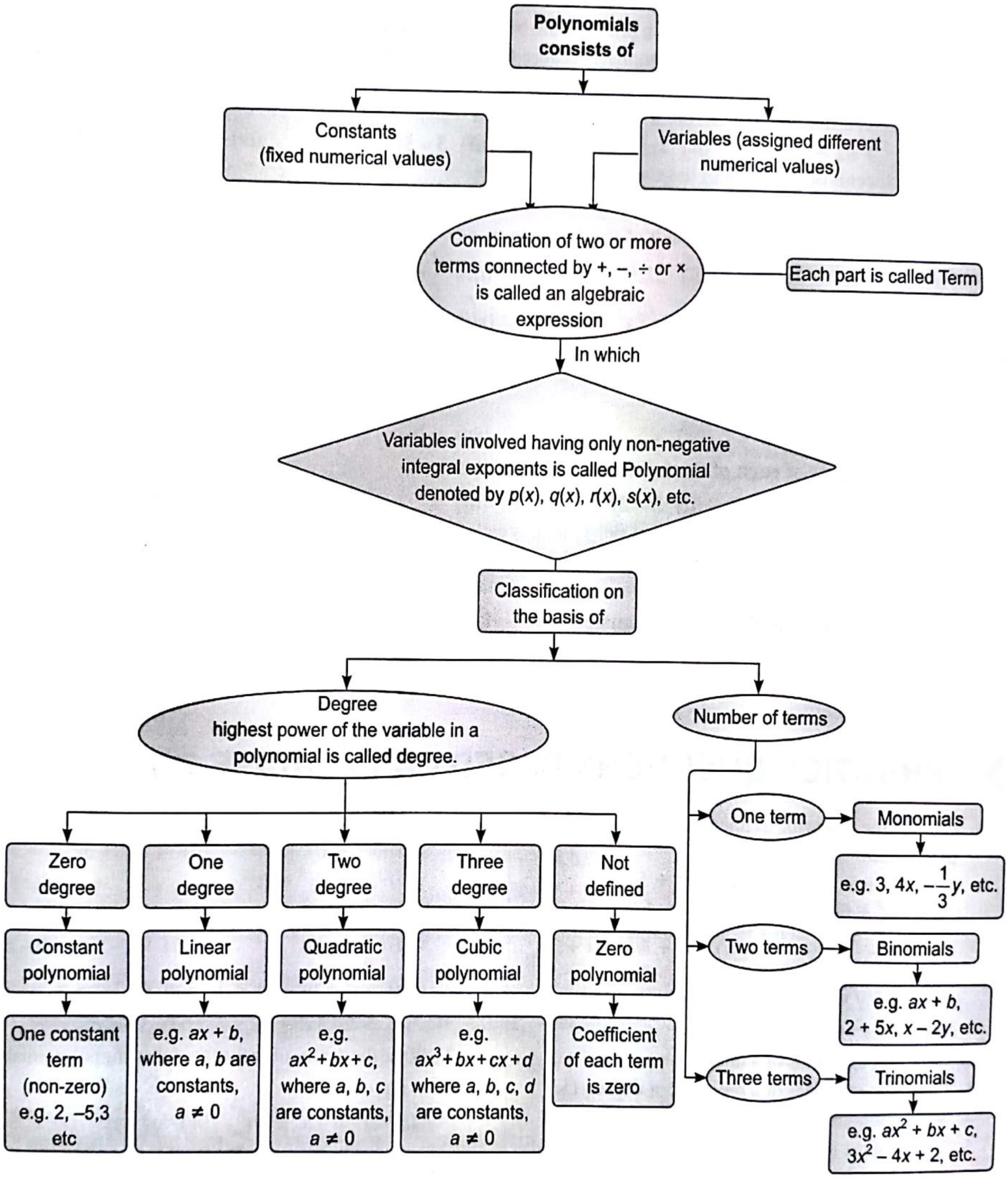
  (i) $(\sqrt[n]{a})^n = a$ (ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ [both a and b should be non-negative integer]

  (iii) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ (iv) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

  (v) $\frac{\sqrt[p]{a^n}}{\sqrt[p]{a^m}} = \sqrt[p]{a^{n-m}}$ (vi) $\sqrt[p]{a^n \times a^m} = \sqrt[p]{a^{n+m}}$ (vii) $\sqrt[p]{(a^n)^m} = \sqrt[p]{a^{n.m}}$

2 POLYNOMIALS

Introduction to Polynomials and Polynomials in one Variable



Zeroes of a Polynomial

Zeroes (Roots) of a polynomial

A number $x = a$ is called zero of the polynomial $p(x)$, if $p(a) = 0$

Can be obtained by

Hit and trial method

By putting $x = a$ in a given polynomial and check
(*i*) If $p(a) = 0$, then 'a' is a zero of the given polynomial.
(*ii*) If $p(a) \neq 0$, then 'a' is not a zero of the given polynomial.

Example:
- Verify whether 3 and 0 are zeroes of the polynomial $2x^2 - 3x$.

Sol. Let $p(x) = 2x^2 - 3x$

Then $p(3) = 2 \times 3^2 - 3 \times 3 = 18 - 9 = 9 \neq 0$

$p(0) = 0 - 0 = 0$

Hence, 0 is a zero of the polynomial $2x^2 - 3x$, but 3 is not.

Solving the polynomial equation by taking $p(x) = 0$

Example: Let $p(x) = 2x + 3$

Its root is given by $p(x) = 0 \Rightarrow 2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

Thus, $x = -\frac{3}{2}$ is a zero of $p(x) = 2x + 3$

Its properties are

- Every linear polynomial has unique zero.
- A non-zero constant polynomial has no zero.
- A polynomial can have more than one zero depending upon its nature.
- Every real number is a zero of the zero polynomial.
- A zero of a polynomial need not be zero.
- 0 may be the zero of a polynomial.

Theorems in Polynomials

Theorems in Polynomlals

Remainder theorem

If $p(x)$ be any polynomial of degree, $n \geq 1$, and 'a' be any real number. When $p(x)$ is divided by linear polynomial $g(x) = (x - a)$ the remainder is $y(x) = p(a)$ and focuses on finding the remainder even if $p(x)$ is not divided by $g(x)$

Example: Find the remainder when the polynomial $p(y) = y^4 - 2y^2 + 3y - 1$ is divided by $g(y) = y - 1$

Sol. Here, the zero of $y - 1$ is 1. By remainder theorem, the remainder is:

$$\begin{aligned} p(1) &= 1^4 - 2 \times 1^2 + 3 \times 1 - 1 \\ &= 1 - 2 + 3 - 1 \\ &= 1 \end{aligned}$$

Factor theorem

If $p(x)$ is a polynomial of degree, $n \geq 1$ and 'a' is any real number, then

(*i*) $x - a$ is a factor of $p(x)$ if $p(a) = 0$ and

(*ii*) $p(a) = 0$ if $(x - a)$ is a factor of $p(x)$

It focuses on

- Checking operation in division of numbers.
- Finding linear factor of $p(x)$.

Example: Determine whether $x - 3$ is a factor of polynomial $p(x) = x^3 - 4x^2 - 3x + 18$

Sol. For $x - 3$ to be a factor of $p(x)$, by factor theorem, $p(3)$ should be zero, where 3 is the zero of $x - 3$.

Now, by reminder theorem,

$$\begin{aligned} p(3) &= 3^3 - 4.3^2 - 3.3 + 18 \\ &= 27 - 36 - 9 + 18 \\ &= 45 - 45 = 0 \end{aligned}$$

Hence, by Factor Theorem, $x - 3$ is a factor of the given polynomial $p(x)$.

Factorisation of polynomials

- Monomials
  - No factor
- Binomials
  - Two perfect squares
  - Two perfect cubes
- Trinomials
  - Quadratic polynomials $p(x) = ax^2 + bx + c$
- Cubic polynomials $p(x) = ax^3 + bx^2 + cx + d$
  - It has on real or distinct real root.
  - One pair of repeated roots and a distinct roots.

Quadratic polynomials $p(x) = ax^2 + bx + c$

1. By middle term spilliting.
   $p + q = b,\ pq = ac$
   (*i*) $ax^2 + bx + c = x^2 + (p + q)x + pq$
   (*ii*) $ax^2 + bx + c = (x + p)(x + q)$
   (*iii*) $ax^2 - bx + c = (x - p)(x - q)$
   (*iv*) $ax^2 \pm bx - c = (x + p)(x - q)$
   Note: In point (*iv*),
   - If b is negative then negative sign goes with bigger number.
   - If b is positive, then negative sign goes with smaller number.
2. If quadratic polynomial is a perfect square, then use
   $x^2 + 2xy + y^2 = (x + y)^2$
   $x^2 - 2xy + y^2 = (x - y)^2$
3. By using factor theorem.

Cubic polynomials — Can be find by using the following steps:
- Write all factors of constant term
- By trial, find $p(q) = 0$
- $(x - q)$ is a factor of $p(x)$
- Divide $p(x)$ by $(x - q)$
- Quotient is a quadratic equation.
- Factorise quadratic equation.

$\therefore\ ax^3 + bx^2 + cx + d \equiv (x - p)(x - q)(x - r)$

Algebraic Identities

Identity I: $(x + y)^2 = x^2 + 2xy + y^2$

Identity II: $(x - y)^2 = x^2 - 2xy + y^2$

Identity III: $x^2 - y^2 = (x + y)(x - y)$

Identity IV: $(x + a)(x + b) = x^2 + (a + b)x + ab$

Identity V: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Identity VI: $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

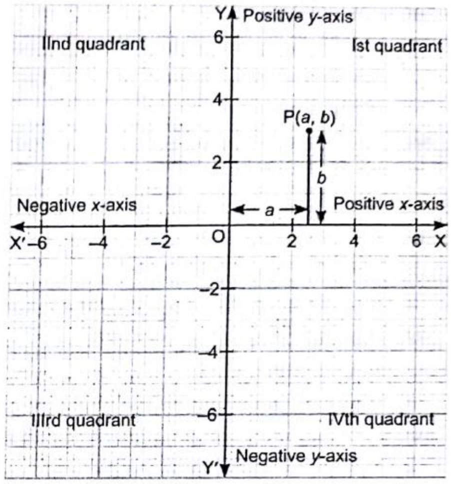
Identity VII: $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
$= x^3 - 3x^2y + 3xy^2 - y^3$

Identity VIII: $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Coordinate Geometry

Coordinate Geometry

- **Coordinate geometry:** It provides a link between algebra and geometry through graphs of lines and curves. This enables the geometrical problems solved algebraically by using coordinate system, i.e. provides a geometric insight into algebra.
- **Cartesian System:** The system used for describing the position of a point in a plane with reference to two fixed mutually perpendicular lines is termed as the Cartesian system.



In a Cartesian system,

(*i*) Horizontal line XX′ is called x-axis, while vertical line YY′ is called y-axis. The two axes XX′ and YY′ are known as the coordinate axes.

(*ii*) The point where these two axes intersect each other is called Origin and is denoted by O.

(*iii*) Since the positive numbers lie on the directions along OX and OY, so OX and OY are called the **positive directions** of the x-axis and the y-axis respectively.

(*iv*) The negative numbers lie on the directions along OX′ and OY′, so OX′ and OY′ are called the **negative directions** of the x-axis and the y-axis respectively.

(*v*) The perpendicular distance from the y-axis measured along the x-axis is called **x-coordinate** or **abscissa**.

(*vi*) The perpendicular distance from the x-axis measured along the y-axis is called **y-coordinate** or **ordinate**.

(*vii*) In stating the coordinates of a point in the coordinate plane, the x-coordinate comes first, and then the y-coordinate. We place the coordinates in brackets, i.e. (x, y).

(*viii*) The order of x and y is important in the coordinate (x, y). So, (x, y) is called an ordered pair. If $x \neq y$, then the ordered pair $(x, y) \neq$ ordered pair (y, x). But if $x = y$, then $(x, y) = (y, x)$.

(*ix*) Origin has zero distance from both the axes so that its abscissa and ordinate both are zero. Therefore, the coordinates of the origin are (0, 0).

(*x*) The coordinates of a point on the positive x-axis are of the form $(x, 0)$ and on the negative x-axis is $(-x, 0)$.

(*xi*) The coordinates of a point on the positive y-axis are of the form $(0, y)$ and on the negative y-axis is $(0, -y)$.

(*xii*) The axes XX′ and YY′ divide the plane into four parts. These four parts are called the **quadrants** numbered I, II, III and IV in anticlockwise direction from OX as shown in the graph.

(*xiii*) **Relationship between the signs of the coordinates of a point and the quadrant of a point in which it lies:**

Quadrant	Abscissa (x-coordinate)	Ordinate (y-coordinate)	Coordinate of the point (x, y)	Reason Quadrant is enclosed by
I	+	+	$(+a, +b)$	positive x-axis and positive y-axis
II	–	+	$(-a, +b)$	negative x-axis and positive y-axis
III	–	–	$(-a, -b)$	negative x-axis and negative y-axis
IV	+	–	$(+a, -b)$	positive x-axis and negative y-axis

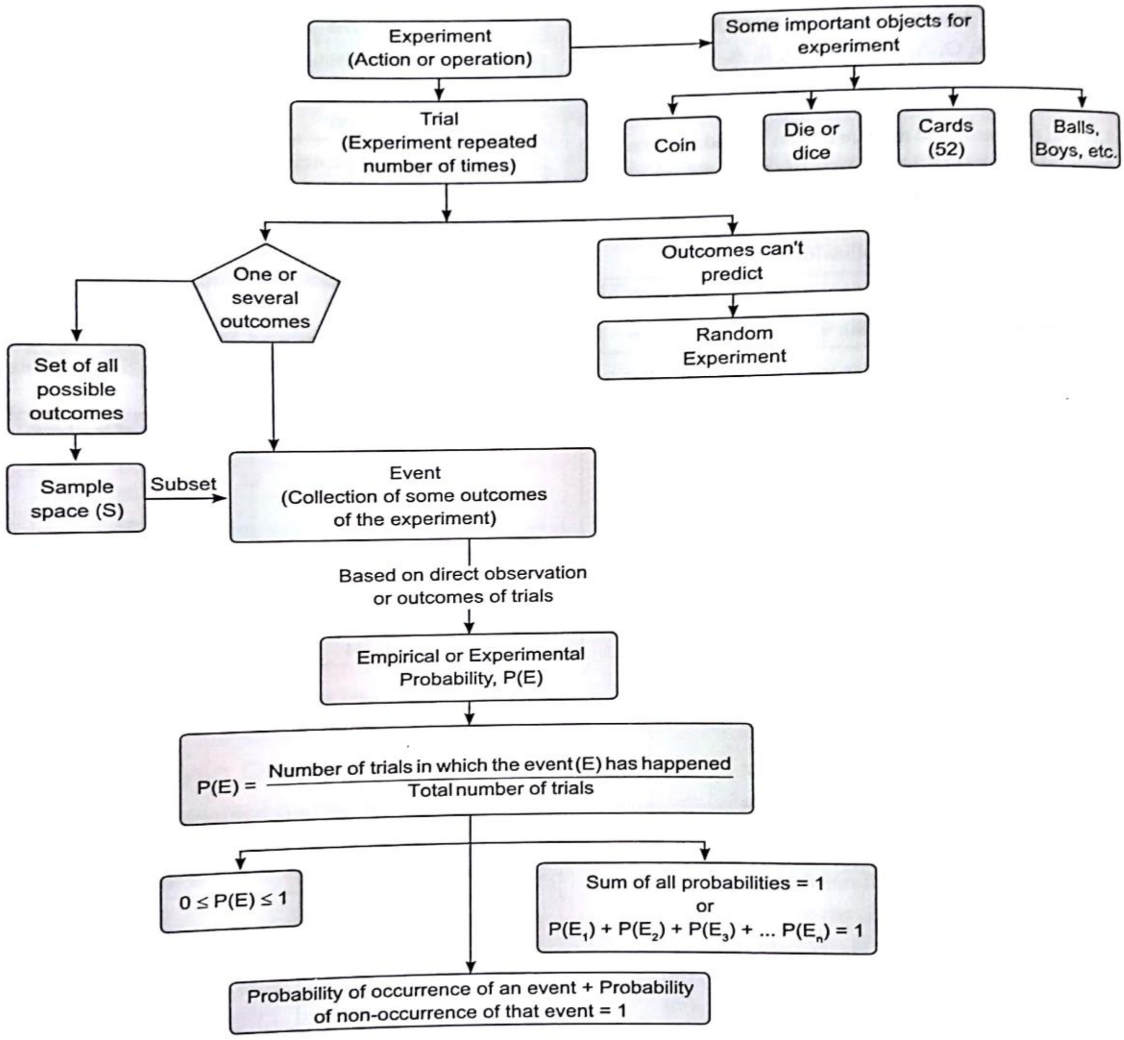
Here, '+' denotes a positive real number, while '–' represents a negative real number.

(*xiv*) The plane consists of the axes and quadrants, are known as **Cartesian plane**, or the **coordinate** plane, or the xy-**plane.**

(*xv*) The equation of x-axis is $y = \mathbf{0}$ and that of y-axis is $x = \mathbf{0}$.

15 PROBABILITY

Probability-an Experimental Approach



- **Number of Experiments:**
  (*i*) When a coin is tossed,

  P(getting a head), $P(H) = \dfrac{\text{Number of heads}}{\text{Total number of trials}}$

  and P(getting a tail), $P(T) = \dfrac{\text{Number of tails}}{\text{Total number of trials}}$

  Also, $P(H) + P(T) = 1$

  (*ii*) When a die is tossed,

$$P(E) = \frac{\text{Number of outcomes having a particular number of die}}{\text{Total number of times the die is rolled (thrown)}}$$

  and $P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) = 1$

  Where $P(E_1)$ = Probability of an event of getting outcome 1.

  $P(E_2)$ = Probability of an events of getting outcome 2 and so on.

Note: • In the similar way, one can find the probability of other experiments.

- Probability of an event can be any fraction from 0 to 1.

$$P(E) + P(\text{not } E) = 1$$

- The empirical (or experimental) probability depends on the number of trials undertaken and the number of times the outcomes occurs in these trials.